Fractal similarity index for forensic handwriting analysis*

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Submitted: October 1, 2013 Accepted: November 26, 2013
**Topic suggested by Dr. Roberto N. Padua, consultant, NORSU and UV. Dr. Padua is currently developing the area of Fractal Statistics to support known results in Fractal Geometry.*

ABSTRACT

The use of fractal analysis is recommended to place handwriting analysis within the purview of science and scientific methods. The current practice of subjecting a questioned document to a handwriting expert's eye borders on art and is replete with subjectivity. A similarity index based on the fractal dimension of a handwritten document is proposed to augment the current practice. A random sample of ten(10) respondents, five(5) male and five(5) female, were asked to copy ten(10) paragraphs of the same lengths by hand. The first paragraphs for each respondent was used as the specimen document while the remaining nine (9) paragraphs were utilized to test and validate the proposed fractal procedure. Results revealed that the similarity index identified authorship as well as genuineness (of signatures) with 99% accuracy.

Keywords: *fractal signatures,sractal analysis, Fractal Geometry, handwriting analysis, questioned document examination*

I. INTRODUCTION

Forged signatures are some of the more common criminal offenses that require careful analysis. Current methodologies used in practice consist of a handwriting expert who, through years of experience, has gain remarkable insights and discrimination ability to distinguish authentic from falsified signatures. Handwriting experts specialize in various aspects of handwriting analysis viz. authenticity assessment, psychological insights based on handwriting, and other facets of handwriting useful in forensic science. Overall, handwriting analysis has remained more of an art than a science. This study attempts to use fractal analysis as a tool for handwriting analysis with focuses on authenticity assessment.

The American Society for Testing and Materials (ASTM) has published standards for many methods and procedures used by Forensic Document Examiner (FDE) (E30.2, 2009 and ASTM Standard E444-09, 2010). An examiner renders scientific examination, comparisons and analysis of documents to: (1) establish genuineness or non-genuineness (expose forgery, reveal alterations, additions or deletion); (2) identity or eliminate persons as the source of handwriting; (3) identity or eliminate the source of typewriting; and (4) write reports or give testimony. There are many historical cases where document examination by FDE's were required: (1) Himmler forged documents (2005); (2) Killian Memos (2004); (3) Martha Stewart Trial (2004); and (4) Anthrax Attack Mailings in the US senate

(2001).

Any document about which some issue has been raised or which is under scrutiny is referred to as questioned document. A disputed document is a questioned document where there is argument over it, usually in terms of ownership. A questioned document, like other documents, may have been prepared with any of the numerous materials available. Sometimes, the very materials of which it is constructed bring discredit and suspicion upon it. Many times, however, its elements are entirely in keeping with its history and purpose, and yet there are those who contest its authenticity. A document may be questioned in whole or in part with respect to its authenticity, identity, origin, relation among its parts, or its relation to other things (Hilton, 1982).

Handwriting identification is another aspect of handwriting analysis which requires identification of authorship. Handwriting identification is a more difficult procedure and requires long study and experience. The problem is that no person ever writes his letters exactly the same way every time. This is apart from the basic principle that "No two people have exactly the same handwriting". The handwriting expert has to learn differences of form and structure by a sort of intuition which was not easily reducible to a science prior to the discovery of fractal geometry by Benoit Mandelbrot (1967/1982) and the subsequent development of fractal statistics by Padua (2012). While no person ever writes his letters exactly the same way every time, there are statistical signatures embedded in that person's fractal writing dimension that can be extracted and exploited as his unique handwriting index.

The main barriers to putting handwriting analysis within the purview of science include: (1) the difficulty in translating handwriting characteristics in mathematical/measurable terms e.g. strokes, slants, form, styles, and others; (2) the difficulty in defining other aspects of handwriting that capture nuances not otherwise earlier identified; and (3) the inherent variances in handwriting that can only be visually detected. Fractal analysis addresses all these barriers because: (1) fractal dimensions measure all geometric forms and patterns that repeat themselves at various scales. They are good and exact measures of the degree of roughness that define all handwriting; (2) visual symbols/representations are immediately amenable to fractal analysis because the technique is rooted in geometry; and (3) fractal dimensions capture almost every characteristic of handwriting, explicit or implicit.

II. CONCEPT OF A FRACTAL AND FRACTAL DIMENSIONS

Classical geometry considers objects that have integral dimensions: points have zero dimensions, lines have one dimension, planes have two dimensions and cubes have three dimensions. Within a plane, one can represent points and straight lines and other geometric objects as shown below:

Figure 1. A fractal object in a plane.

It is possible to represent geometric objects within a plane that are neither points nor lines like the squiggly line above. This squiggly geometric object cannot have dimension equal to 1 because it fills up more space than a line; it cannot have dimension equal to 2 because it does not form an area. Hence, its dimension λ has to be between 1 and 2 like $\lambda = 1.63$. We will say that the squiggly line is a fractal (a geometric object having fractional dimension).

The fractal dimension of an object defines its roughness, ruggedness or fragmentation. The higher the fractal dimension, the more rugged and irregular-looking the object is. Thus, although fractals are rough and irregular objects, the pattern of irregularities are repeated over and over again. This is called the self-similarity property of fractal. Benoit Mandelbrot (1967/1982) is acknowledged as the mathematician who opened roughness as a legitimate topic for investigation in modern science. He claimed that nature and natural processes are fractals, while uniform, smooth and continuous patterns are man-made concepts and pervade mathematical analysis. He also said that by introducing "randomness" into the situation, one gets more realistic fractal representations.

After the publication of Mandelbrot's book: Fractals: The Geometry of Nature, many scientists used fractals with great success: (1) Cohen, (1995) on fractal antennae; (2) Krummel (1986) on forest fractals and others). It has found applications in various disciplines as well as in many areas of practical technology.

Handwriting analysis, a forensic science discipline, is one area where the power of fractal analysis can be put to bear. The handwritten text can be viewed as "symbols of repeated strokes" and, hence, fractals. Each individual possesses a unique fractal writing signature that, in some sense, distinguishes his writings from everybody else. It is this visual sense of uniqueness of individual handwritings that can be tested for fractality and this is what we will exploit in this paper. As explained earlier, the current state of the art in handwriting analysis is largely subjective and is more of an art than a science. The fact that each person's handwriting contains individual differences is the fundamental principle on which handwriting comparisons are based. The natural and subconscious handwriting characteristics developed by an individual are a product of both the movements of the hand and the mind which directs the writing (Ordway, 1982).

Writing characteristics involves form, system, muscular habits/coordination, straight lines, curves, angles, proportion, line quality, retracing, connections, size, slant, spacing, strokes, and others. All these characteristics give the appearance of ruggedness and roughness which, we contend, can be summarized in a single number – the fractal dimension (λ).

In Padua (2012), fractal geometry was translated to statistical language. A probability distribution akin to Pareto's distribution for incomes was proposed as a model for fractal random variables X:

(1) $f(x) = \frac{(\lambda-1)}{\theta}\left(\frac{x}{\theta}\right)^{-\lambda}, x \geq \theta, \lambda > 0$

Where λ = fractal dimension of x, $\theta = \inf_x\{x\}$. A maximum – likelihood estimator for λ based on a random sample of size n was provided as:

(2) $\hat{\lambda} = 1 + n\left(\sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right)\right)^{-1}$.

He then proceeded to show that for n=1:

(3) $z = \hat{\lambda}\log\left(\frac{x}{\theta}\right) - 1 \underset{\sim}{d} Exp(\lambda - 1)$ or:

(4) $q(z) = (\lambda - 1)\, exp(-(\lambda - 1)z)$

For a random sample of size n, the random variable:

(5) $q = \hat{\lambda}\sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right) - n$

Has the same distribution as $q^* = \sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right) = \sum_{i=1}^{n} Z_i$ The distribution of (5) is therefore $Gamma\left(n, \beta = \frac{1}{\lambda-1}\right)$ where $\lambda > 1$:

(6) $h(q) = \frac{(\lambda-1)^n}{\Gamma(n)} q^{n-1} e^{-q(\lambda-1)}, q > 0, \lambda > 1$.

$$h(q) = \frac{(\lambda-1)^n}{\Gamma(n)!} q^{n-1} e^{-q(\lambda-1)}$$

Thus, if we have one person's handwriting and if we are able to estimate his (geometric) fractal from this handwriting (see for example some available freeware like FRAK.OUT), then we are able to compare the fractal dimension for the questioned document (say, λ_1) with his specimen handwriting (λ_2):

(7) $u = |\lambda_1 - \lambda_2|$.

We approximate the distribution of u by an exponential distribution and obtain:

(8) $\delta_s = P(u \geq \varepsilon) = \frac{1}{2}\left(1 + exp\left(-\varepsilon^{(\lambda_2 - 1)}\right)\right)$, a similarity index

where λ_2= fractal dimension q specimen handwriting. We refer to (8) as a similarity index. As the difference $\varepsilon = |\lambda_1 - \lambda_2|$ increases, the similarity index decreases. If $\lambda_1 = \lambda_2$ (hence, $\varepsilon = 0$), the fractal dimensions are identical and the two documents are 100% similar. This means that the two documents contains exactly the same writing characteristics: straight lines, curves, strokes, spacings, slants and so on, and, must therefore have been written by the same author.

It is also possible to determine what values of ε will yield high similarity index thus:

(9) $\delta_s \geq 1 - \alpha \Leftrightarrow \varepsilon \leq \left[\log \frac{1}{1-2\alpha}\right]\left[\frac{1}{\lambda_2 - 1}\right], 0 \leq \alpha \leq 1$

For instance, if $\alpha = 0.05$, then the values of ε above will indicate 95% similarity index or greater.

III. RESEARCH DESIGN AND METHODS

The study is designed to assess the viability of using fractal analysis in determining (a) authorship; and/or (b) genuineness of a handwritten document. As such, two distinct procedures are adopted corresponding to these two objectives.

For the problem of determining authorship of a handwritten document, ten (10) respondents, five (5) male and five (5) female, were requested to copy by hand, ten (10) paragraph consisting of equal number of words. The first paragraph is considered the specimen handwriting of the respondents. The nine (9) other paragraphs are then compared with the specimen handwriting using the similarity index (8). For each respondent, we calculated:

(10) $$\begin{gathered}\textit{Percent Correct Identification (PCI)}\\ \textit{Per Respondent}\end{gathered}$$

$$= \frac{\textit{No. of } \delta_s \textit{ greater or equal to } 99\%}{9} x100\%$$

The PCI's are then compared across sexes to determine if the proposed methodology is sensitive to gender differences.

On the other hand, the same methodology and formula (10) are used to authenticate signatures of the respondents. To augment the handwriting metric (10), we also computed for average similarity index (δ_s) per respondent:

(11) $$\textit{Average Similarity Index Per Respondent}$$

$$= \frac{\sum_{i=1}^{q} \delta_{s_i}}{q}$$

IV. RESULTS AND DISCUSSIONS

Table 1 shows the fractal dimensions of the handwritten paragraphs by the respondents. The fractal dimensions were obtained by the box-counting method.

Table 1. Fractal Dimensions of Handwritten Paragraphs by Respondents

sex	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10
0	1.9662	1.9787	1.9691	1.9701	1.9802	1.9553	1.9778	1.9692	1.9699	1.979
1	1.9491	1.9357	1.9451	1.925	1.9676	1.9491	1.9367	1.9449	1.925	1.9676
0	1.9583	1.9635	1.9593	1.9706	1.9672	1.9583	1.9635	1.9593	1.968	1.967
0	1.6002	1.6142	1.5699	1.5699	1.6039	1.623	1.6142	1.63	1.5699	1.6039
0	1.9781	1.9835	1.9676	1.9676	1.9522	1.9781	1.9835	1.9676	1.9479	1.953
1	1.591	1.5832	1.6047	1.5205	1.6199	1.5444	1.5832	1.6047	1.5325	1.6199
1	1.9782	1.9539	1.9758	1.9922	1.9771	1.9782	1.9539	1.9771	1.9922	1.9771
0	1.5689	1.566	1.5704	1.5407	1.5281	1.5931	1.566	1.581	1.5407	1.5281
1	1.9414	1.9687	1.9659	1.9611	1.9624	1.9414	1.9687	1.961	1.9611	1.9624
1	1.9804	1.9796	1.9686	1.9809	1.9881	1.9804	1.9796	1.979	1.9809	1.9881

Table 2 shows the calculated deviations of the fractal dimensions of each of the questioned documents (P2-P10) from the assumed specimen document (p1).

Table 2. Distance(ε) of specimen handwriting (P1) from questioned documents P2-P10.

eps2	eps3	eps4	eps5	eps6	eps7	eps8	eps9	eps10
0.0125	0.0029	0.0039	0.014	0.0109	0.0116	0.003	0.0037	0.0128
0.0134	0.004	0.0241	0.0185	0	0.0124	0.0042	0.0241	0.0185
0.0052	0.001	0.0123	0.0089	0	0.0052	0.001	0.0097	0.0087
0.014	0.0303	0.0303	0.0037	0.0228	0.014	0.0298	0.0303	0.0037
0.0054	0.0105	0.0302	0.0259	0	0.0054	0.0105	0.0302	0.0251
0.0078	0.0137	0.0705	0.0289	0.0466	0.0078	0.0137	0.0585	0.0289
0.0243	0.0024	0.014	0.0011	0	0.0243	0.0011	0.014	0.0011
0.0029	0.0015	0.0282	0.0408	0.0242	0.0029	0.0121	0.0282	0.0408
0.0273	0.0245	0.0197	0.021	0	0.0273	0.0196	0.0197	0.021
0.0008	0.0118	0.0005	0.0077	0	0.0008	0.0014	0.0005	0.0077

Table 3 shows the computed similarity indices based on Equation (8).

Table 3: Similarity Indices of questioned documents (P2-P10) from specimen (P1).

index2	index3	index4	index5	index6	index7	index8	index9	index10	MEAN SIMILARITY INDEX	PCI
0.99	1.00	1.00	index5	0.99	0.99	1.00	1.00	0.99	0.9963	100%
0.99	1.00	0.99	index6	1.00	0.99	1.00	0.99	0.99	0.9941	100%
1.00	1.00	0.99	index7	1.00	1.00	1.00	1.00	1.00	0.9974	100%
1.00	0.99	0.99	index8	0.99	1.00	0.99	0.99	1.00	0.9935	100%
1.00	0.99	0.99	index9	1.00	1.00	0.99	0.99	0.99	0.9929	100%
1.00	1.00	0.98	index10	0.99	1.00	1.00	0.98	0.99	0.9910	100%
0.99	1.00	0.99	index11	1.00	0.99	1.00	0.99	1.00	0.9951	100%
1.00	1.00	0.99	index12	0.99	1.00	1.00	0.99	0.99	0.9950	100%
0.99	0.99	0.99	index13	1.00	0.99	0.99	0.99	0.99	0.9907	100%
1.00	0.99	1.00	index14	1.00	1.00	1.00	1.00	1.00	0.9986	100%

Table 4 shows the analysis of variance performed to determine if the fractal dimensions of the handwritten documents can be differentiated between respondents.

Table 4. Analysis of Variance of Fractal Dimensions Between Respondents
One-way ANOVA: R1, R2, R3, R4, R5, R6, R7, R8, R9, R10.

Source	DF	SS	MS	F	P
Factor	9	3.151464	0.350163	1123.54	0.000
Error	90	0.028049	0.000312		
Total	99	3.179514			

Individual 95% CIs For Mean
Based on Pooled StDev

Level	N	Mean	StDev	-------+---------+---------+---------
R1	10	1.9716	0.0077	*)
R2	10	1.9446	0.0149	(*
R3	10	1.9635	0.0045	(*)
R4	10	1.5999	0.0226	(*
R5	10	1.9659	0.0146	(*)
R6	10	1.5804	0.0359	*)
R7	10	1.9756	0.0130	(*
R8	10	1.5583	0.0225	(*)
R9	10	1.9594	0.0099	(*
R10	10	1.9806	0.0054	(*)
				-------+---------+---------+---------
Pooled StDev =	0.0177			1.65 1.80 1.95

TUKEY'S POST-HOC: 5.08, p<.0001

Table 5. Correlations: R1, R2, R3, R4, R5, R6, R7, R8, R9, R10.

	R1	R2	R3	R4	R5	R6	R7	R8	R9
R2	0.212								
	0.556								
R3	0.551	-0.183							
	0.099	0.613							
R4	-0.016	0.402	-0.478						
	0.965	0.250	0.162						
R5	-0.193	-0.046	-0.745	0.594					
	0.594	0.899	0.013	0.070					
R6	0.505	0.784	-0.292	0.385	0.161				
	0.137	0.007	0.414	0.273	0.657				
R7	-0.423	-0.111	0.319	-0.571	-0.782	-0.434			
	0.224	0.761	0.369	0.084	0.007	0.211			
R8	-0.666	-0.202	-0.874	0.474	0.784	-0.102	-0.326		
	0.035	0.575	0.001	0.167	0.007	0.779	0.358		
R9	0.782	-0.182	0.478	-0.188	-0.183	0.236	-0.369	-0.398	
	0.008	0.615	0.162	0.602	0.613	0.511	0.294	0.255	
R10	0.387	0.472	0.529	0.293	-0.386	0.132	0.111	-0.614	-0.095
	0.269	0.169	0.116	0.411	0.271	0.717	0.759	0.059	0.793

Cell Contents: Pearson correlation (p-value)

V. DISCUSSIONS

It is noted that indeed the fractal dimensions of the paragraphs written by each respondent vary but the observed variations are within 1% of the fractal dimension of the specimen handwriting. This supports the principle that "*no person ever writes his letters exactly the same way every time*."

However, the similarity indices computed per individual for each of the questioned documents 2 to 10 registered values well beyond 99%. That is, each of the documents written by an individual has 99% or more similarity with his specimen document. Moreover, the individual PCI's (percent correct identification) were all 100% i.e. the fractal method correctly identified a questioned document as having been written by the individual respondent.

The analysis of variance performed on the fractal handwriting dimensions of the ten (10) respondents yielded a very significant f-value of 1123.4 exceeding the required value for significance at the 0.01 probability level. This means that no two individuals have the same fractal handwriting dimensions, a fact that is supported by a post-hoc analysis using Tukey's method (Average Tuk = 5.08, p<.0001). Similarly, the correlation matrix (Table 5) revealed that either the respondents' handwritings were negatively correlated (opposite in terms of roughness) with each other or bear positive similarity but not significant either at .05 or .01 probability levels. Again, this result exemplifies the principle that "*No two people have exactly the same handwriting*".

These results imply that the fractal handwriting dimension (FHD) of an individual serves as his unique handwriting index. This index is similar to the more current DNA testing for physical evidences found in crime scenes. Since individual DNA's are unique, they can be used for purposes of identifying a crime perpetrator or for eliminating suspects.

Table 6. Fractal dimensions of respondents' signatures.

sex	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10
0	1.9808	1.9225	1.9524	1.9548	1.9564	1.9393	1.9395	1.9789	1.9387	1.962
1	1.9422	1.926	1.9067	1.9593	1.9344	1.9588	1.9588	1.9193	1.9609	1.9399
0	1.9917	1.975	1.975	1.9062	1.9796	1.9796	1.9899	1.9011	1.9877	1.9789
0	1.6221	1.6246	1.613	1.6371	1.6351	1.6276	1.6333	1.625	1.6231	1.622
0	1.9232	1.9267	1.9406	1.9287	1.9477	1.9067	1.9226	1.9222	1.9231	1.9222
1	1.5726	1.5995	1.58	1.5788	1.5766	1.5701	1.5699	1.5733	1.572	1.6001
1	1.9655	1.9655	1.9307	1.962	1.9306	1.9577	1.97001	1.9643	1.9655	1.97
0	1.4498	1.4531	1.3989	1.4338	1.455	1.4489	1.4497	1.4398	1.4411	1.45
1	1.9503	1.9349	1.9337	1.9502	1.9511	1.9512	1.9409	1.962	1.9671	1.9561
1	1.9541	1.9049	1.9668	1.9263	1.9374	1.977	1.954	1.9544	1.9542	1.9547

Table 7. Similarity Indices of signatures from specimen.

index1	index2	index3	index4	index5	index6	index7	index8	index9	MEAN SIMILARITY INDEX
0.97	0.99	0.99	0.99	0.98	0.98	1.00	0.98	0.99	0.98
0.99	0.98	0.99	1.00	0.99	0.99	0.99	0.99	1.00	0.99
0.99	0.99	0.96	0.99	0.99	1.00	0.96	1.00	0.99	0.99
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.00	0.99	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00
0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00
1.00	0.98	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00
1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.99	0.99	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00
0.98	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	0.99

Table 8. Analysis of variance for differentiating signatures.

Source	df	SS	MS	F	P
Factor	9	3.548274	0.394253	1240.41	0.000
Error	90	0.028606	0.000318		
Total	99	3.576880			

Level	N	Mean	StDev	*Individual 95% CIs For Mean Based on Pooled StDev* -----+---------+---------+---------+-
r1	10	1.9525	0.0184	(*)
r2	10	1.9406	0.0191	*)
r3	10	1.9665	0.0336	(*)
r4	10	1.6263	0.0072	*)
r5	10	1.9264	0.0112	*)
r6	10	1.5793	0.0113	*)
r7	10	1.9582	0.0149	(*
r8	10	1.4420	0.0165	(*)
r9	10	1.9498	0.0108	(*)
r10	10	1.9484	0.0206	(*)
				-----+---------+---------+---------+-
Pooled StDev = 0.0178				*1.50 1.65 1.80 1.95*
Tukey = 6.09, p < .0001				

VI. DISCUSSIONS

Variations in the fractal dimensions of the signatures of the respondents are noted as well. Of the ten(10) respondents, respondent 1 registered the highest variations. As in the previous analysis, results tend to support the principle that individuals write differently every time but the variations in their signatures are well within 1% to 2% from each other.

Except for respondent 1, all the similarity indices are 99% or higher. For respondent 1, however, the mean similarity index was computed at 98% . The lower similarity index for this particular respondent is due to the higher variations observed in his signatures. The fractal handwriting dimension (FHD) remains valid as a unique indicator of a person's signature.

The analysis of variance performed on the fractal dimensions of the individuals' signatures revealed that the respondents' signature can be significantly differentiated from each other (f-value = 1240). The computed f-ratio exceeded the required value for significance beyond the 0.01 probability level.

VII. PROPOSED PROTOCOLS FOR HANDWRITING ANALYSIS AND QUESTIONED DOCUMENT EXAMINATION

While the fractal handwriting dimension index (FHD) does give unique signatures for different individuals, it is possible to incorrectly attribute a handwritten document to an individual who is not the author of the document. For instance, respondents 5 (fractal dimension = 1.9781) and 7 (fractal dimension = 1.9782) have almost the same specimen handwriting. For this reason, we propose the following protocols:

Protocols:

1. For handwritten specimen documents having more than one paragraphs (or sentences), obtain the fractal dimensions of the paragraphs (or sentences). Use this fractal dimensions as

the specimen fractal dimension.

2. Do the same for the questioned document.
3. Compute the correlation coefficient between the fractal dimensions of the questioned documents and the specimen fractal dimension.

Illustration:

Suppose that Respondent 3 claims he is the author of a document (actually written by Respondent 1). The fractal dimensions of the paragraphs written by Respondents 1 and 3 are displayed below together with the computed correlation coefficient:

Figure 8. Respondents 1 and 3 with the Computed Correlation Coefficient.

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1.9662	1.9787	1.9691	1.9701	1.9802	1.9553	1.9778	1.9692	1.9699	1.979
1.9583	1.9635	1.9593	1.9706	1.9672	1.9583	1.9635	1.9593	1.968	1.967

pearson correlation of R1 and R3 = 0.551
p-value = 0.099

We claim that there is no significant relationship between the two fractal dimensions and, thus, Respondent 3 cannot claim ownership of the document and we are 90.1% confident of this conclusion.

Next suppose, that Respondent 7 claims that he is the author of a document actually written by Respondent 5, we perform a similar analysis:

Figure 9. Respondents 7 and 5 with the Computed Correlation Coefficient.

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1.9781	1.9835	1.9676	1.9479	1.9522	1.9781	1.9835	1.9676	1.9479	1.953
1.9782	1.9539	1.9758	1.9922	1.9771	1.9782	1.9539	1.9771	1.9922	1.9771

pearson correlation of R5 and R7 = -0.782 (p-value = 0.007)

The fractal dimensions are negatively related so that the handwritings of the two individuals are the exact opposite of each other. There is a significant evidence that Respondent 7 cannot claim authorship of the document ($p<.01$) and we are 99% confident of this conclusion.

Finally, suppose that Document 1 is being questioned for authorship and there are nine (9) claimants , namely R2 to R10. Who is the most likely author of the document?

We compute the correlation matrix and determine the most positively related fractal dimensions with Document 1. The claimant who is the most likely author of the document is R9:

pearson correlation of R1 and R9 = 0.782
p-value = 0.008
r-squared value = 61.15%

The fractal handwriting dimension of R9 explains about 61.15% of the variations in the fractal handwriting dimensions of the questioned document Document 1. We are about 99.2% confident of this decision to assign authorship to R9.

VIII. CONCLUSION

The proposed fractal handwriting dimension (FHD) index is a unique measure of an individual's handwriting and signature. It can be used as a method to augment current practices for determining (a) authorship; and/or (b) genuineness of handwritten documents.

Originality Index:	93 %
Similarity Index:	7 %
Paper ID:	384252431
Grammarly:	Checked

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